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Spectator interactions and factorization in $B \to \pi l \nu$ decay

M. Beneke[1] and T. Feldmann[2]

[1] Institut für Theoretische Physik E, RWTH Aachen, 52056 Aachen, Germany
[2] CERN Theory Division, 1211 Geneva 23, Switzerland

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Abstract. We investigate the factorization of different momentum modes that appear in matrix elements for exclusive B meson decays into light energetic particles for the specific case of $B \to \pi$ form factors at large pion recoil. We first integrate out hard modes with virtualities of order m_b^2 (m_b being the heavy quark mass), and then hard-collinear modes with virtualities $m_b \Lambda$ (Λ being the strong interaction scale). The resulting effective theory contains soft and collinear fields with virtualities Λ^2. We prove a previously conjectured factorization formula for $B \to \pi$ form factors in the heavy quark limit to all orders in α_s, paying particular attention to 'endpoint singularities' that might have appeared in hard spectator interactions.

PACS. 12.39.St Factorization – 13.20.He Decays of bottom mesons

1 Introduction

In recent years considerable progress in the theoretical description of exclusive B meson decays into light energetic particles has been achieved. The 'QCD-factorization approach' [1] allows for a systematic computation of short-distance QCD corrections in the heavy quark limit. It can also be embedded into the soft-collinear effective field theory (SCET) framework [2,3,4,5]. Factorization proofs for this class of B decays are complicated, because two hard scales, m_b and $\sqrt{m_b \Lambda}$, are involved by interactions with the spectators in the B meson. Simpler exceptions are the decays $B \to \gamma \ell \nu$ [6,7,8] (where collinear dynamics is absent at leading power) and $B \to D\pi$ [9,10] (where spectator interactions are absent), for which factorization proofs to all orders in perturbation theory have been given in the heavy quark limit.

In this article we focus on the superficially simplest 'non-trivial' case, namely the factorization theorem for $B \to \pi$ form factors:

$$F_i(q^2) = C_i(q^2)\, \xi_\pi(q^2) + \phi_B \otimes T_i(q^2) \otimes \phi_\pi + \text{power-suppressed terms} \qquad (1)$$

proposed in [11]. Here $\xi_\pi(q^2)$ denotes a single form factor independent of the Dirac structure $\bar{q}\Gamma_i b$ of the weak decay current. It reflects the approximate large-recoil symmetry relations for heavy-to-light form factors [12]. Corrections to these symmetry relations arise from radiative corrections to the decay vertex and hard spectator scattering. The coefficient functions $C_i(q^2)$ and hard-scattering kernels $T_i(q^2)$ are calculable perturbatively in α_s. In addition, the factorization formula involves the (non-perturbative, but process-independent) light-cone distribution amplitudes ϕ_B and ϕ_π of the B meson and pion.[1] The present status of the factorization theorem (1) is as follows.

- The short-distance functions $C_i(q^2)$ and $T_i(q^2)$ have been determined to order α_s [11], confirming the structure of (1).
- In SCET the functions $C_i(q^2)$ are interpreted as operator-matching coefficients, which arise from integrating out hard modes (virtualities of order m_b^2) [2]. The renormalisation group in SCET can be used to sum large logarithms $\ln m_b$.
- The hard-scattering kernels $T_i(q^2)$ are obtained by integrating out hard-collinear modes (see Table 1 below) that arise from interactions of the external soft and collinear fields, and have virtualities of order $m_b \Lambda$. In this way the tree-level expressions for $T_i(q^2)$ have been recovered [15], and the general procedure for a two-step matching procedure in SCET to all orders in perturbation theory has been outlined.

A proof of (1) to all orders in α_s requires a detailed analysis of the second matching step, in which hard-collinear modes are integrated out perturbatively, since only then can the problem of convergence of the soft and collinear convolution integrals in (1) be addressed. Possible 'endpoint divergences' are related to the fact that the resulting operators in the effective theory do not necessarily

[1] The first term in (1) resembles the well-known factorization formula for $B \to D$ decays where the form factors factorize into perturbative coefficient functions and a universal Isgur–Wise form factor [13]. The second term in (1) is analogous to the factorization formula for the pion form factor at large momentum transfer [14].

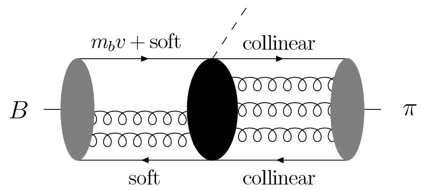

Fig. 1. Kinematics for $B \to \pi$ form factor at large recoil

factorize into purely soft or collinear hadronic quantities in the conventional sense. To prove (1), we have to define $\xi_\pi(q^2)$, and show that the remainder factorizes into conventional distribution amplitudes convoluted with a hard-scattering kernel $T_i(q^2)$, free of endpoint singularities.

In the following we sketch the main steps that lead to the completion of the factorization proof (for details see [16]). The effective theory set-up is similar to [15]: we perform the matching in two steps, onto an effective theory in which the degrees of freedom are quark and gluon fields with virtualities of order Λ^2. In the rest frame of the B meson, the initial state consists of only soft modes with all momentum components of order Λ. The final state pion is made up only of collinear modes (Fig. 1). Hard and hard-collinear modes have been integrated out and are encoded as perturbative coefficient functions multiplying effective operators. Table 1 summarizes the various modes and the scaling of the momentum components defined with respect to two light-cone vectors $n_\pm$ (the pion momentum is $p_\pi = En_-$, and for the heavy quark velocity we take $v_\perp = 0$). The expansion parameter in the effective theory is $\lambda = \sqrt{\Lambda/m_b}$.[2]

Table 1. Scaling of momentum modes relevant to $B \to \pi$ form factors. The entries in brackets denote an alternative terminology, which can be found in the literature

$(n_+p, p_\perp, n_-p)$	terminology	(alternative)
$(1,1,1)$	hard	(hard)
$(1,\lambda,\lambda^2)$	hard-collinear	(collinear)
$(\lambda^2,\lambda^2,\lambda^2)$	soft	(ultrasoft)
$(1,\lambda^2,\lambda^4)$	collinear	(ultracollinear)

2 Results

We first checked the relevance of the different modes in Table 1 by considering a toy loop integral with the same kinematics as in $B \to \pi$. The expansion of the integral is reproduced by momentum regions along the lines of [17], including separate hard-collinear and collinear regions. The collinear integral suffers from an endpoint divergence (not regularized in d dimensions), when the n_+k component of the loop momentum becomes small. This divergence is cancelled by an endpoint divergence in the soft region, arising from $n_-k \to 0$. The relation between endpoint divergences in soft and collinear integrals prevents the naive factorization of the form factor into light-cone distribution amplitudes (in which case we could have set $\xi_\pi(q^2)$ in (1) to zero).

Turning to the construction of the effective theory, we first integrate out hard modes at a factorization scale $\mu \sim m_b$, and arrive at an effective theory SCET(hc,c,s), where soft and collinear fields interact by exchange of hard-collinear modes (heavy quarks only interact with soft modes). The first step consists of classifying all current operators in SCET(hc,c,s), which contribute to the leading-power form factor. To this end we determine the additional λ suppression factors in the matrix elements of SCET(hc,c,s) operators incurred by the necessity to convert hard-collinear fields into collinear and soft fields via power-suppressed interaction terms. We then find only two possible structures [4,18]:

$$\tilde{C}_{ij}\left(\frac{\mathcal{E}}{m_b},\frac{\mu}{m_b}\right)\bar{\xi}_{\rm hc}(0)\Gamma_j h_v(0), \tag{2}$$

$$\frac{1}{m_b}\int d\hat{s}\;\tilde{C}'_{ij\perp}\left(\hat{s},\frac{\mathcal{E}}{m_b},\frac{\mu}{m_b}\right)\bar{\xi}_{\rm hc/c}(0)A^\perp_{\rm hc}(sn_+)\Gamma_j h_v(0), \tag{3}$$

where $\mathcal{E} = (n_-v)(n_+P)/2$, and P is the (collinear) momentum operator. In the second line the quark field can be collinear or hard-collinear. We also adopt the gauge $n_+A_{\rm hc/c} = n_-A_s = 0$. The gauge transformations that undo light-cone gauge introduce collinear and soft Wilson lines into the operators. The projection properties of the ξ and h_v fields imply that $\Gamma_j = \{1,\gamma_5,\gamma_\perp^\mu\}$ is the most general Dirac structure. The short-distance coefficients from integrating out hard modes may depend only on the boost-invariant and dimensionless ratios $\mathcal{E}/m_b$, μ/m_b and $\hat{s} = m_b s/(n_-v)$. Note that in SCET(hc,c,s) the operator (3) is suppressed relative to (2), but both contribute to the leading-power form factor [15].

We now define the $C_i(q^2)\xi_\pi(q^2)$ term in (1) as the matrix element of the operator (2). Since only $\Gamma_j = 1$ does not vanish between $\langle\pi|$ and $|B\rangle$, this defines one non-perturbative form factor independent of the original Dirac structure. It remains to show that the second operator (3) factorizes into light-cone distribution amplitudes (LCDAs). Although we do not need to further discuss the operator (2) at this point, we note that if one attempted to factorize it into LCDAs, the resulting convolution integrals would be divergent. Furthermore, three-particle LCDAs of the B meson and pion would appear even at leading power (as also noted in [19]). We also find that the matrix element of (2) scales as λ^3, which confirms the well-known scaling of heavy-to-light form factors at large recoil.

[2] The only objects scaling with an odd power of λ are soft quark fields and meson states. Because B mesons contain an even number of soft quark fields, the expansion parameter for exclusive matrix elements in the effective theory is in fact $\lambda^2 = \Lambda/m_b$, but the absolute scaling may be an odd power of λ.

In the second matching step hard-collinear modes are integrated out at $\mu \sim \sqrt{m_b \Lambda}$. The resulting effective theory is denoted by SCET(c,s). The following results and arguments are relevant to the factorization proof:

• Soft and collinear fields are decoupled in the relevant terms of the SCET(c,s) Lagrangian.

• Two-quark operators in SCET(c,s) do not contribute to the hard-scattering amplitudes $T_i(q^2)$, because the collinear (soft) fields in the operator must have the same quantum numbers as the pion (B meson). The relevant terms in the matching of (3) on SCET(c,s) operators must therefore contain the fields $[\bar{\xi}_c \xi_c][\bar{q}_s h_v]$, and possibly additional gluon or quark fields with the correct quantum numbers.

• Dimensional analysis and boost invariance imply that the operator (3) matches only onto four quark operators at leading power,

$$\begin{aligned}&\bar{\xi}_{\mathrm{hc/c}}(0) A^{\perp}_{\mathrm{hc}}(sn_+) \Gamma_j \, h_v(0)\\ &\to \int ds' dt \, \tilde{C}''_{jk}\big(\hat{s}; \ln[\mu^2 s' t], n_+ P s'\big)\\ &\qquad \times [\bar{\xi}_c(s' n_+) \Gamma_k h_v(0)] \, [\bar{q}_s(t n_-) \Gamma_k \xi_c(0)]. \qquad (4)\end{aligned}$$

The right-hand side scales as λ^8, implying a contribution to the form factor at order λ^3, which is as large as the $C_i(q^2)\xi_\pi(q^2)$ term. We also note that the effective current (4) must be local in the transverse direction.

• The right-hand side of (4) has the required structure $\phi_B \otimes T_i(q^2) \otimes \phi_\pi$ after taking the $B \to \pi$ matrix element, but it remains to show that the convolution integrals converge (under the assumption of the standard endpoint behaviour of the LCDAs). Boost invariance implies that the short-distance coefficients $\tilde{C}''_{jk}$ may depend only logarithmically on the light-cone distance t between the heavy quark and the soft spectator, and that only the plus-projection of the B meson light-cone matrix element appears in (4). After taking the $B \to \pi$ matrix element, the t-convolution integral is therefore

$$\int dt \, \tilde{C}''_{jk}(\ln t) \, \tilde{\phi}^B_+(t). \qquad (5)$$

This integral converges, given the endpoint behaviour of the B meson light-cone distribution amplitude $\tilde{\phi}^B_+(t)$ [20].

• It follows that the collinear convolution integrals over $\hat{s}$ and s' also converge. If this were not the case, the collinear convolution integral would have to be regulated. The regulator dependence introduced by the endpoint divergence would have to cancel against a soft endpoint divergence (since the effective theory is assumed to reproduce the infrared physics correctly). Since there is none, the collinear convolution integrals must be finite.

This completes the proof of the factorization formula (1). Both terms in the formula are of order λ^3, as far as heavy quark power counting is concerned. The hard scattering term is proportional to $\alpha_s(\sqrt{m_b \Lambda})$. It is an open question whether the $C_i(q^2)\xi_\pi(q^2)$ term is proportional to 1 (as assumed in [2,11,12]), or proportional to $\alpha_s(\sqrt{m_b \Lambda})$. The clarification of this point requires an analysis of large logarithms in SCET(c,s).

3 Summary and outlook

We have proved the factorization formula (1) for $B \to \pi$ transition form factors in the heavy quark limit. Attempting to factorize heavy-to-light form factors along the same lines as the $\pi \to \pi$ transition form factor [14], it is found that three-particle Fock states of the B meson and the pion contribute at leading power, and that the convolution integrals are in general ill-defined. We have shown that all this is part of a single form factor $\xi_\pi(q^2)$, while the remaining contributions can be factorized in the standard way. This confirms the factorization conjecture of [11] and completes the factorization argument of [15]. Our result generalizes to other heavy-to-light form factors ($B \to \rho$ etc.). We remark that in the case of the $B \to \gamma$ form factor, the endpoint divergences and 'non-factorizable' contributions only appear at subleading order in the heavy quark expansion, when the hadronic structure of the photon is resolved. Our framework can also be adapted to situations with two collinear directions, such as for the pion electromagnetic form factor at large momentum transfer. For details we refer to a forthcoming paper [16].

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